
Tree It: A Tangible User Interface for Constructing the Sample Space

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Abstract

UPDATED—April 30, 2018. In math curriculum, probability is one of the most difficult concepts for students to understand [7]. With the intention of making abstract math concepts more concrete, we designed Tree It to facilitate third and fourth grade students' construction of the sample space for probabilistic events through the visualization of a tree diagram. With a constructionist perspective in mind, our design incorporates a tangible user interface (TUI) along with customizable tangibles and activities. This gives learners the ability to explore concepts of permutations through systematic enumeration by moving tangible objects.

Author Keywords

basic probability, tangible user interface, permutations, mathematics education

ACM Classification Keywords

H.5.2 [Information interfaces and presentation]: User Interfaces; K.3.1 [Computers and Education]: Computer Uses in Education

Introduction

Probability is a difficult concept to intuitively grasp. Consequently, we want to introduce basic probability to younger students through a constructionist point of view. Tree It is

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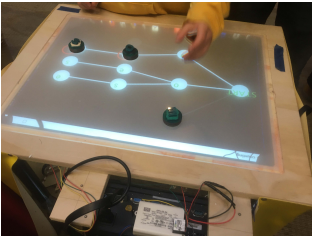


Figure 1: Student using Tree It.

a tangible user interface (TUI) platform designed to introduce third and fourth grade students to probabilistic thinking. With the support of the graphical interface and a guiding classroom activity, learners construct a tree diagram by moving three tangible objects (Figure 1) that represent the permutable items. In each depth of the tree, nodes represent a potential placement of an object. Placing a correct object at a node sprouts new branches from the node representing subsequent nodes. Once all nodes of the tree are filled, all permutations are found and the sample space is exhausted.

Tree It is intended to be a multimedia platform and a classroom tool. The learning goal is for students to concretize the sample space through a means of systematically and exhaustively generating possibilities, from Horvath and Lehrer's [8] learning framework for teaching probability. The tree structure serves to aid this goal by highlighting patterns between elements of the sample space.

Background

Thinking about probability is often characterized as unintuitive because it is different from most of school mathematics, where students think stochastically rather than discretely.

Probabilistic Thinking

It is a common phenomena among children as well as adults to apply deterministic explanations to situations in probability that involve chance [9], which may be a result of a larger cultural bias towards deterministic explanations in formal education [7]. As a result, it is often the case that students are "limited to plugging numbers into formulas for permutations and combinations without developing any conceptual understanding" [2]. However, to learn probability, students must reconcile their primary schemata with

reasoning that is specific to stochastic situations [6]. We aim to design a technology that addresses this issue starting from basic probabilistic principles.

Theoretical Rationale

We aim to approach basic probability through a constructionist perspective [11] by having students explore the topic in personally meaningful ways. The combination of a TUI (Figure 2) and tangibles allows for learners to express ideas, such as the sample space, through different media [3] the sample space. The objects can move independently and simultaneously on the table, allowing for multiple students to construct knowledge collectively and collaboratively on an externalized shared object so the "mind is no longer to be located entirely inside the head" [4]. The choice to represent three objects is intentional; it is a relatively simple case of permuting objects without being trivial, so it introduces students to a "specific, well-chosen case" to understand learning. As Papert explains, students can later "generalize from this understanding" [11]. Additionally the interface environment serves as a microworld, where students have the agency to explore the tree construction and debug their understanding between abstract and concrete [11] by moving objects on the table.

Related Work

In a study by Manches, children were asked find all the ways to additively create a selected number and those who used tangibles identified more solutions than those who used pencil and paper [10]. This further motivates our usage of tangible objects in math education.

Other work like Combinatorix and Seeing Chance address teaching probability with tangibles. Combinatorix also helps students explore, solve and understand combinatorics problems through a TUI table but for high school students [13]. Our system targets the learning of similar concepts, but for

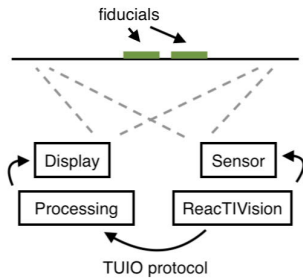


Figure 2: Diagram of TUIO interface.

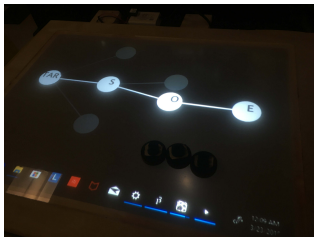


Figure 3: Creating a single permutation in the form of a branch. Gray nodes are the cloud states.

a younger age group and a greater emphasis on the tree representation. Seeing Chance presents differentiating outcomes in a compound event space with students in a similar age range (4th and 6th graders) [1]. Our project takes a step back from differentiating specific examples to being able to enumerate all examples in the sample space for our slightly younger age group.

Design

Tree It was designed in response to students struggling with concepts of probability. The intended use is for a student or students to use Tree It to explore problems structured by scripted activities in the presence of a guiding teacher. We are in the beginning stages of development and have thus far created Tree It independent of user input with our next steps being user testing. It is our intention to provide users with a stand-alone product and observe how users interact with Tree It in order to develop future features with their responses in mind.

Tree Structure

In many college-level probability classes instructors make use of tree diagrams to illustrate the sample space. Tree It applies this powerful visualization and exposes probabilistic thinking to younger students. A tree diagram provides affordances for children to explore and find patterns within the sample space. The branch structure of our design serves as a scaffold for students to explore possible outcomes. The design of the cloud nodes (Figure 3) represent potential future pathways with their gray color; gray having the visual connotation of being in between letting students know they are close to discovering the next node. Users have the freedom to order objects as long it is valid, which allows for multiple correct representations of the sample space.

We have identified two main systematic strategies for users

to enumerate the same space: building the tree one stage at a time and one branch at a time. In both strategies, the user develops an understanding of how the tree is built on the past decisions. By building the tree, the user can develop a large-scale picture of how a tree of “n” elements grows. The structure of the tree naturally lends itself to systematic organization and enumeration of the sample space.

Tangible User Interface

For our TUI, we used ReactTVision to detect fiducials, and coded it using Processing and the TUIO application. Physically, there is a table with a camera to track the fiducials adhered to the bottom of the objects and a projector to create the table image (Figure 2). The TUI provides cognitive offloading by maintaining a record of previous permutations on the screen. After the user has placed a tangible on a valid cloud node, the user can remove the tangible and observe the node’s change in state from cloud (unknown) to white (defined). This record of already taken paths allow the user to focus on finding patterns and the systematic creation of new branches. Additionally, the TUI offers immediate feedback for users. Tree It models permutation without replacement. Thus, the table prevents repetitive sequences as well as repetitive letters within a sequence by not accepting repeated objects in a branch. The lack of cloud nodes indicates to students if all the possibilities are exhausted on a local (Figure 3) or global (Figure 5) scale.

Classroom Activities

In our goal of making math more relevant and meaningful, we have developed activities, in which students are given real-world situations that facilitate probabilistic thinking. For example, students are asked to decode a secret password from a string of letters or identify possible arrangement of items for a storefront. Tree It becomes a valuable tool for students to translate their ideas onto the screen, to keep a



Figure 4: Tangibles used to mark where the permutable objects belong in the tree. These are customized to fit the activities.

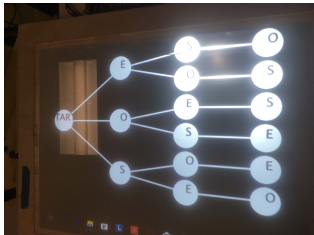


Figure 5: Completed tree on the TUI table.

record of their “identified” permutations, and to recognize the potential paths to complete the tree.

Tangibles

The tangibles are laser cut and 3D printed objects that assist in completing the activities and mapping the tree. Each tangible has a unique fiducial attached to them, representing the specific permuting object. The users can attach the permuting object with putty to the base of a tangible or place the object directly into one of the cuplike tangibles. We created a kit of tangibles (Figure 4) to accompany our example classroom activities, but teachers and students could easily create activities related to objects that student create on their own or bring from home.

Future Work

In our current iteration of Tree It, students can build out a complete tree on the TUI. However after initial user testing, we want to design for activities that use the completed tree (Figure 5) to emphasize the patterns and qualities of an exhausted sample space. This could look like identifying common substrings or creating an activity around regenerating the individual branches from the final nodes. For our final demo at the conference, we plan to enhance the tactile or visual properties of the tangible, so users can intuitively know that the fiducials are designed to be moved around perhaps by taking inspiration from cultural forms that are used to leave behind markings like stamps. We also would consider creating richer scripts that surround tree formation and additional activities with the tree structure that could foster more productive learning and demonstrate positive learning effects [5, 12].

Conclusion

Tree It, a tangible user interface, provides an environment for introducing young students to probabilistic thinking us-

ing their own customized and culturally relevant objects. Through the movement of tangibles, the students simultaneously construct the branches of the tree and build the sample space in both systematically and exhaustively. We build on existing platforms [13, 1], but address an age group not targeted as much by designers (third and fourth grade students). We aim to promote the use of visual and tangible representations in mathematics education.

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