# DiMBI: An Interface to Connect People to Math's Big Ideas of Patterns and Relations

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## Abstract
Math education faces the challenge not only of *how* to teach math, but also of *what* math to teach. With the intention of promoting math's big ideas in formal and informal settings, we created DiMBI (Discovering Math's multimedia platform developed in Processing that uses reacTIVision to read users' interactions with selected tangible regular polygons (a triangle and a square). The corners of each polygon are linked with specific features that aim to promote transfer learning (colors and music). The features change as the user manipulates the objects through symmetric actions and permutation of corners. This is the first stage of our technology, which aims to bring big math ideas closer to the people, through more concrete interactions.

**Figure 1**: The current version in upper portion of the TUI screen.


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## Categories and Subject Descriptors
H.5.2 [Information Interfaces and Presentation]: User Interfaces. K.3.1 [Computers and Education]: Computer Uses in Education

## Introduction
DiMBI provides a multimedia platform that allows students to weave together combinatorial patterns with music and color to provide a rich, personalizable experience of exploration. By combining physical manipulatives with virtual feedback we created an environment for students to reflect about and externalize their thinking processes [5].

DiMBI inhabits a unique niche that connects combinatorics (the study of *how many ways*) and abstract algebra (the study of epistemic structures, of what makes two structures identical, of ways to construct and re-present structures), while scaffolding freedom of exploration, discovery, and expression as students manipulate and create their own audio-visual experience.

## Background
This project is driven by the notion that providing children multimedia tools to discover big ideas can re-empower both the big ideas and the children who re-discover them [5]. We focus

### Learning Objectives

- *Pattern exploration*

  Students will gain familiarity with symmetries and permutations as manipulatable objects.

- *Rigid motion*

  Students will learn to generate symmetries through rotations and reflections of the square and triangle.

- *Isomorphism*

  Students will understand that permutation of elements (swapping polygon vertices) leads to a set of symmetries with identical structure.

- *Transfer/Creation*

  Students will be able to transfer the big idea of patterns into two parallel pattern-based elements: music and color, and will be able to create their own audio-visual experience by applying reflections, rotations, and permutations to the physical shapes.

on centering one of math's biggest ideas: patterns and relations. Developing a deep understanding of patterns and relations is foundational not only to mathematics, but also to computer science, physical science, and most forms of problem solving [3] and critical thinking.

Research on student engagement with patterns has identified that what students notice affects both what sorts of exploration and what type of thinking they are able to engage in [4]. Digital technologies provide the possibility of scaffolding what a student notices as they freely explore.

*Tangibles*

Incorporating tangible objects into multimedia educational environments can support student engagement in playful learning and can facilitate collaborative interaction with a concept [2, 6]. Tangible user interfaces have been used in math to support learning in a wide variety of topics. TanTab creates a multimedia environment for students to play with Tangrams and develop spacial reasoning skills [2]. Pattern blocks have been used to help children explore symmetry [8], but only with respect to axes, and with no connection to grouping or numeric representation of symmetries. Furthermore, these math technologies are confined to visual representations of the concepts at hand.

*Multiple Representations*

A powerful math technology should legitimate different forms of mathematical understanding via developing integrated, dynamic representations. In the case of patterns, the possibility for students to represent (with visual, auditory, and numeric representations) different forms of the same idea through the use of multimedia tools can also deepen their understanding. [7]

*Math and Music*

Bamburger has done substantial research on music education and the role of music as embodied mathematics [1]. However, to our knowledge, there have not been technologies developed using music to underscore mathematical patterns such as symmetries and permutations. The role of music in DiMBI is not only as an aid to gaining mathematical understanding, but also as means of expressing mathematical understanding that has been gained. Evidence of a student's deep understanding of a big idea is the appropriate transfer of that idea into different contexts. We thus see the musical element of DiMBI as a chance for students to concretely express their pattern discoveries as an auditory representation.

## Design

DiMBI has been designed primarily for students from ages 10-14. Driving our design decisions are the four key learning objectives laid out at the left.

*Design Principles*

After choosing the core mathematical concepts, educational theories, and learning goals that sit at the heart of our project, we defined and applied the following three user principles.

1. *PHYSICAL INTERACTION*.

During the process of prototyping different DiMBI designs, we recognized the importance of having physical objects connected to a screen interface. A 10 year old participant said: *"I like this better than the computer because I can manipulate it."* Physical manipulation of an object to gain intuition remains a core part of DiMBI.

2. *ASSISTED FLEXIBILITY*.

One of the core characteristics of DiMBI is that it allows students to freely explore symmetries of rotations, reflections, and transpositions using pre-built polygons.

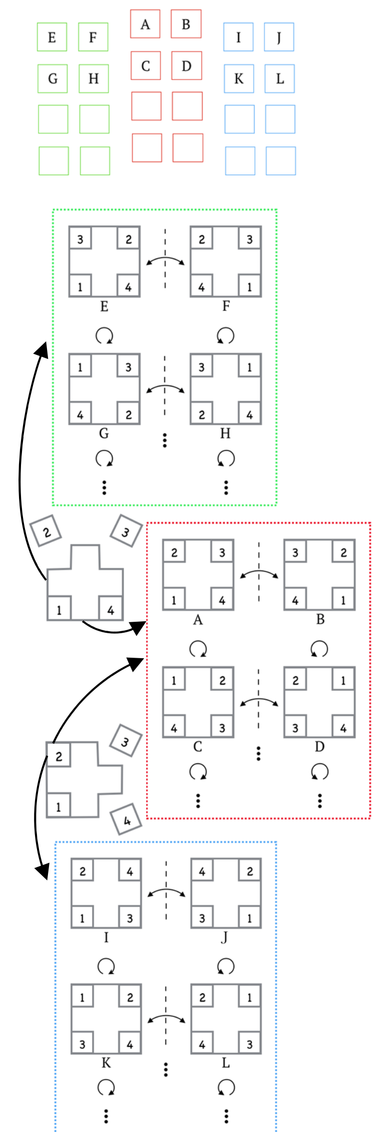


**Figure 2**: How manipulations (rotations, reflections, and corner transpositions) correspond to the squares in upper portion of the TUI screen.

Currently, there are two types of regular polygons in our design: triangle and square. Students have the flexibility to freely explore with the tool to better notice the symmetries and permutations, while having assistance from DiMBI to guarantee that learning objectives are accomplished. To ensure adherence to this principle, DiMBI has limited what types of interactions students can have with the technology (they are limited to *rigid motion symmetries* i.e., rotation and reflection, and to *permutations* i.e., taking off corners and swapping them). However, the symmetries they choose to place down, the order in which they create those symmetries, and the representations they notice (musical, color, numeric) are all up to the user.

3. *CONCRETE KNOWLEDGE EXPRESSION*.
With the intention of making the abstract ideas of symmetries and permutations more concrete, each polygon's corner is associated with a number (visible on the physical corner and on the corresponding location of virtual screen when the object is placed down) and with a unique percussive sound. When the polygon is set down, its nodes are played on a 12-beat measure, on a continuous loop. Applying a symmetry action or a permutation to the shape produces the associated sequence change in the numbers and sounds of the shape's corners.

The upper portion of the TUI screen (Figure 1) has six small triangles (arranged as 3x2) and three sets of eight small squares (arranges as 4x2).[1] Each of these four sections is associated with a set of polygon permutations closed under symmetry actions, with a hue, and with a musical instrument. By permuting corners, students can access different colors/instruments. By performing symmetric actions within a section, students can change the brightness and saturation of the hue along with the chord produced by the instrument. (See Figure 2 for a summary of sample manipulations.) The changes in musical tones and in hues help students to identify what happens when they change the positions of each of the corners, and to express their knowledge through creation of a desired compilation of chords and colors.

*Hardware*
The tangibles in DiMBI are laser cut wooden polygons whose corners have unique fiducials fastened to them. These corners can be a removed from the polygon, swapped with other vertices, and slid back into place, kept in position by a loose variation of the mortise-tenon joint. With this design, depicted in Figure 4, rotation and reflection of the objects are the easiest forms of manipulation. The user experiences a bit heavier cognitive load when manipulating vertices, to cue her attention. With this, we believe our design prompts the user to question whether this type of vertex manipulation (a permutation) might be fundamentally different from the other forms of manipulation (symmetry via rotation and reflection), which is then underscored by the corresponding change in color and instrument.

The TUI table incorporates a camera to read in the fiducials and a projector to create the tabletop images seen by the user, both incorporated beneath the tabletop.

*Software*
The virtual environment was created using Processing, a Java-based programming language. Our primary two reasons for selecting this open-source language were its strong built-in visualization environments and its compatibility with reacTIVision. The reacTIVision software uses a computer vision

[1]For the mathematical reader: these correspond to the dihedral group $D_6$ and the three isomorphic copies of $D_8$ within $S_4$.

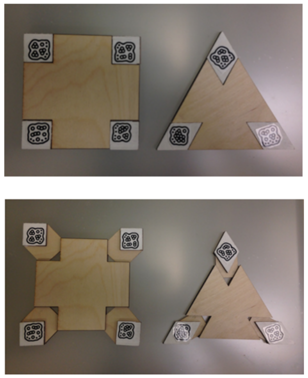

**Figure 3**: The current physical manipulatives used in DiMBI.

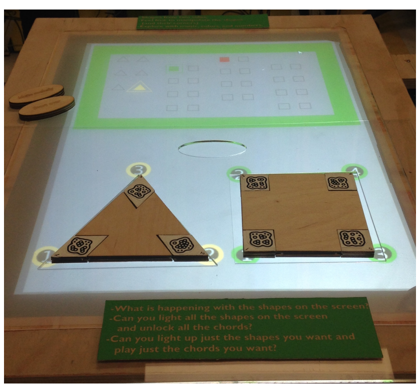


**Figure 4**: The current version of DiMBI.

algorithm to collect local information about each of the fiducials that enters the camera's scope.

## Future Work

This first version of DiMBI promotes the successful accomplishment of the learning goals. In the short term, we plan to find other physical mechanisms similar to those within a Rubik's cube to promote our assisted flexibility principle. We believe this added structure might assist users in developing and testing meaningful schemata, thereby facilitating deeper understanding of the learning objectives. Once we have this mechanism, we plan to conduct user test sessions in formal and informal learning environments during three stages: interviews, think alouds, and focal groups. In the future, we hope to incorporate more freedom and personalization in choosing what audio and visuals are associated with each component of the interface. We also hope to incorporate a recording system in future iterations, so students can listen their thinking process used to arrive to different solutions. Additionally, future versions of DiMBI could usefully mediate a socially engaged and extrinsically motivated context. In a second phase, we plan to implement an interface that allows collaboration between pairs.

## Conclusion

DiMBI was designed as an object-to-think-with, an "object in which there is an intersection of cultural presence, embedded knowledge, and the possibility for personal identification." [5] Through externalizing (via music, color, and physical manipulatives) the often abstract notions of permutations and symmetric actions, students are able to concretize the big idea of pattern generation. While the technology requires additional testing and further developments, it appears to be a promising tool for promoting big math ideas in informal or classroom settings.


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